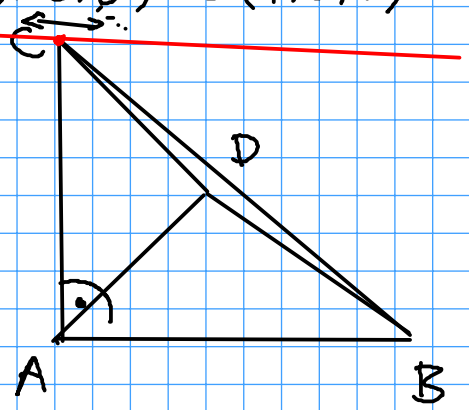


Aufgabe 1 A(3/6/10) B(4/8/8) P(-1/3/8) Q(4/3/6)

a) $g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 9 \\ 8 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

$g := \begin{bmatrix} -1+5s \\ 9-6s \\ 8-2s \end{bmatrix}$ $\downarrow \downarrow r :=$



$\vec{BA} \cdot \vec{CA} = 0$ $a := \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$ $b := \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$

Solve (dotp(a-b, g-a) = 0, s)

$\begin{pmatrix} 3-4 \\ 6-8 \\ 10-8 \end{pmatrix} \cdot \begin{pmatrix} -1+5s-3 \\ 9-6s-6 \\ 8-2s-10 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -4+5s \\ 3-6s \\ -2-2s \end{pmatrix} = 0$

$-1(-4+5s) - 2(3-6s) + 2(-2-2s) = 0$

$4 - 5s - 6 + 12s - 4 - 4s = 0$

$-6 + 3s = 0$

$s = 2$

$\vec{OC} = \begin{pmatrix} -1 \\ 9 \\ 8 \end{pmatrix} + 2 \cdot \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \\ 4 \end{pmatrix}$ C(9/-3/4)

b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$ ← D liegt da

$\|\vec{DA}\| = \|\vec{DB}\|$

TR: norm(d-b)

$d := \begin{bmatrix} 4+5s \\ 8-6s \\ 8-2s \end{bmatrix}$ $d := b + s \cdot r$

$\left\| \begin{pmatrix} 4+5s-3 \\ 8-6s-6 \\ 8-2s-10 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 5s \\ -6s \\ -2s \end{pmatrix} \right\|$

norm(a-d) = norm(b-d)

$\sqrt{(1+5s)^2 + (2-6s)^2 + (-2-2s)^2} = \sqrt{(5s)^2 + (-6s)^2 + (-2s)^2}$

Solve(\uparrow , s) $\Rightarrow s = \frac{3}{2}$
enter

$d | s = \frac{3}{2}$ ($\frac{3}{2}$ für s einsetzen)

$D \left(\frac{23}{2} \mid -1 \mid 5 \right)$

c)

$\vec{n} = \vec{AB} \times \vec{AD}$

$\vec{n} = \begin{pmatrix} 4 & -3 \\ 8 & -6 \\ 8 & -10 \end{pmatrix} \times \begin{pmatrix} 1 \cdot 5 & -3 \\ -1 & -6 \\ 5 & -10 \end{pmatrix} =$

$\begin{pmatrix} 1 & & & 8.5 \\ 2 & \times & & -7 \\ -2 & & & -5 \\ 1 & & & 8.5 \\ 2 & & & -7 \end{pmatrix} = \begin{pmatrix} 2 \cdot (-5) - (-2) \cdot (-7) \\ -2 \cdot (8.5) - 1 \cdot (-5) \\ \dots \end{pmatrix}$

$d := \begin{bmatrix} 23 \\ 2 \\ -1 \\ 5 \end{bmatrix}$

$n := \text{crossp}(b-a, d-a)$

$\vec{n} = \begin{pmatrix} -24 \\ -12 \\ -24 \end{pmatrix}$

$n := \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

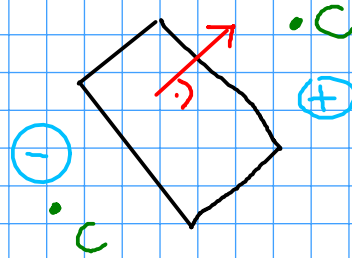
Ebene: $2x + y + 2z = d$

$2x + y + 2z - 32 = 0$

A einsetzen:

$2 \cdot 3 + 6 + 2 \cdot 10 = 32$

HNF: $\frac{2x + y + 2z - 32}{\sqrt{2^2 + 1^2 + 2^2}} = 0$

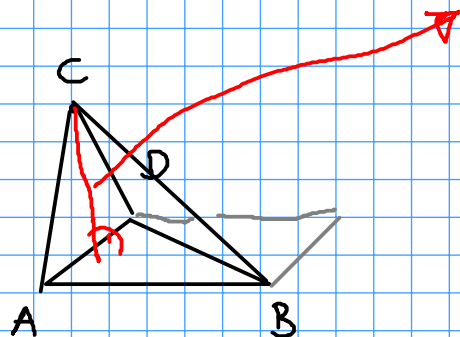


Abstand: C in HNF einsetzen

$\frac{2 \cdot 9 + (-3) + 2(4) - 32}{3} = \frac{-9}{3} = -3 \Rightarrow \text{Abstand: } \underline{\underline{3}}$

d) $V = \frac{1}{3} G \cdot h$

$G = \frac{\|\vec{AD} \times \vec{AB}\|}{2}$



$$\text{crossp}(d-a, b-a) \begin{pmatrix} 24 \\ 12 \\ 24 \end{pmatrix}$$

$$\text{norm}(\quad) \rightarrow 36$$

$$G = 18$$

$$V = \frac{1}{3} \cdot 18 \cdot 3 = \underline{\underline{18}}$$

Aufgabe 6

a) $(x|0|0)$ $(0|y|0)$ $(0|0|z)$

$$3x = 24$$

$$4y = 24$$

$$12z = 24$$

$$x = 8$$

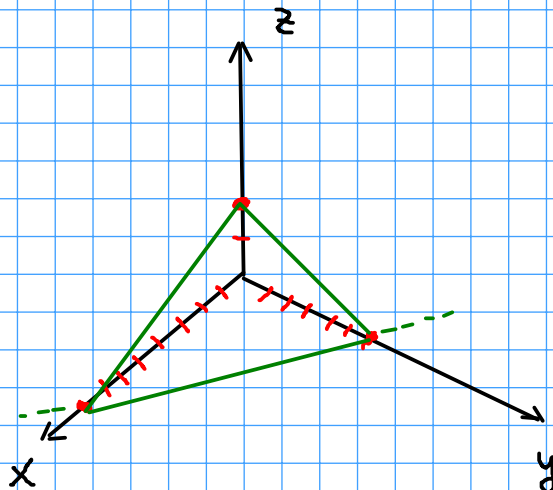
$$y = 6$$

$$z = 2$$

$$(8|0|0)$$

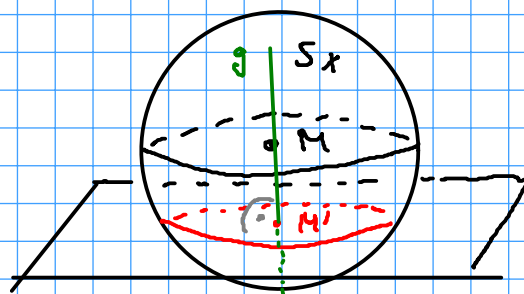
$$(0|6|0)$$

$$(0|0|2)$$



- d) g geht durch M
 g ist senkrecht zu E

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5.5 \\ 2 \\ 7 \end{pmatrix} + s \cdot \begin{pmatrix} 3 \\ 4 \\ -12 \end{pmatrix}$$



$$3x + 4y + 12z - 24 = 0$$

$$3(5.5 + 3s) + 4(2 + 4s) + 12(7 + 12s) = 24$$

$$16.5 + 9s + 8 + 16s + 84 + 144s = 24$$

$$169s = -84.5$$

$$s = \frac{-84.5}{169} = -\frac{1}{2}$$

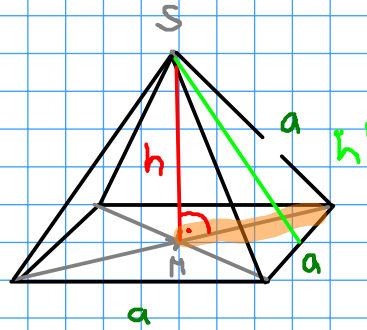
$$\vec{OM'} = \begin{pmatrix} 5.5 \\ 2 \\ 7 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

$$\underline{\underline{M'(4|0|1)}}$$

e) $h = \|\vec{SM}\|$

$$\vec{SM} = \begin{pmatrix} 5.5 - 8.5 \\ 2 - 6 \\ 7 - 19 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ -12 \end{pmatrix}$$

$$\sqrt{(-3)^2 + (-4)^2 + (-12)^2} = 13$$



$$d = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2} a$$

$$\frac{\sqrt{2}}{2} a$$

$$13^2 + \left(\frac{\sqrt{2}}{2} a\right)^2 = a^2$$

$$13^2 + \frac{2}{4} a^2 = a^2$$

$$13^2 = \frac{1}{2} a^2$$

$$2 \cdot 13^2 = a^2 \quad |\sqrt{\quad}$$

$$\sqrt{2} \cdot 13 = a$$

$$h' = \sqrt{13^2 + \left(\frac{\sqrt{2} \cdot 13}{2}\right)^2}$$

$$= \sqrt{13^2 + \frac{2 \cdot 13^2}{4}}$$

$$= \sqrt{\frac{4 \cdot 13^2 + 2 \cdot 13^2}{4}}$$

$$= \sqrt{\frac{6 \cdot 13^2}{4}} = \frac{\sqrt{6} \cdot 13}{2}$$

$$O = G + 4 \cdot F_{\Delta} = 2 \cdot 13^2 + 4 \frac{a \cdot h'}{2}$$

$$= 2 \cdot 13^2 + 2 \cdot \sqrt{2} \cdot 13 \cdot \frac{\sqrt{6} \cdot 13}{2} = 2 \cdot 13^2 + \sqrt{12} \cdot 13^2$$

$$= (2 + 2\sqrt{3}) 13^2 = \underline{\underline{923.43}}$$